Analysis of Algorithms and Heuristic Problem Solving, 2021/22, 01 September 2022
All questions count equally. Literature, electronic and communication devices are not allowed. It is allowed to use 1 sheet of A4 format paper. You can write your answers in either English or Slovene. Duration: 90 minutes.
Students who wish to look into the written exam results can do so on Friday, 02 September 2022, at 12:00 in the room of Prof Robnik Šikonja (2nd floor, room 2.06).

1. Find the solution to the recurrence
   $T(n) = 2T\left(\frac{n}{15}\right) + T\left(\frac{n}{10}\right) + 2T\left(\frac{n}{6}\right) + \sqrt{n}$

2. Consider the following randomized algorithm for computing the smallest element in an array which is randomly shuffled.

```
RandomMin(A[1 .. n]) {
  min ← ∞
  for i ← 1 to n
     if A[i] < min
       min ← A[i]    // ⋆
  return min ;
}
```

   a) In the worst case, how many times does RandomMin execute line marked with ⋆?
   b) What is the probability that line marked with ⋆ is executed during the n^{th} iteration of the for loop?
   c) What is the exact expected number of executions of line marked with ⋆?

3. Describe the similarities and differences between the fireworks algorithm and firefly algorithm in optimization. In which way they both implement the intensification strategy? Justify your answer.

4. You are given a task to solve the facility assignment problem defined as follows. There is a set *U* of users (defined with locations) that need access to a service, and a set of possible server locations *S*. For each site $s \in S$, there is a fee $f_s \geq 0$ for placing a server at that location. Users $u \in U$ can be served from multiple sites, with associated cost c_{us} for serving user *u* from site *s*. If cost c_{us} is high, we will avoid serving user u from site s; in this way we can promote serving users from nearby sites.
   For sets *U* and *S*, and cost functions *f* and *c*, you have to select a subset $A \subseteq S$ at which to place servers and assign each user to the active server where it is cheapest to be served. i.e. you have to minimize the overall cost $\sum_{s \in A} f_s + \sum_{u \in U} \min_{s \in A} c_{us}$.
   Propose features to be used in solving the problem with the guided local search.