1. It's a sunny April day and Victor is craving for a beer upon returning home. Problem: There is not a single beer can in the fridge. He quickly puts a beer can (which is at 24°C at the moment) in the fridge (which is constantly at 4°C) and waits for half an hour. Once he gets the beer out of the fridge, it has 14°C. (Victor keeps an infrared thermometer always handy at home...)
   (a) Write down and solve the differential equation, which determines the temperature of the beer can depending on time.
   Hint: The cooling rate is proportional to the difference in temperatures.
   (b) How long should Victor keep the beer in the fridge to cool it down to 9°C?

2. Find the general solution of the differential equation

$$y' = 2x(1 + y^2),$$

   and the solution satisfying the condition $y(1) = 0$.

3. Find the general solution of the *logistic differential equation*

$$y' = cy\left(1 - \frac{y}{a}\right),$$

   and the solution satisfying the condition $y(0) = b$.

4. Write an Octave function `[t, Y] = euler(f, [t0, tk], y0, h)`, which solves the differential equation

$$y' = f(t, y) \quad \text{with initial condition} \quad y(t_0) = y_0$$

   using the Euler method with step size h. The function should return a set of function values `Y` evaluated at times `t`.

   Solve DE's above using this method. Compare exact and numerical solutions.

5. Write an Octave function `[t, Y] = rk4(f, [t0, tk], y0, h)`, which solves the differential equation

$$y' = f(t, y) \quad \text{with initial condition} \quad y(t_0) = y_0$$

   using the classical order 4 Runge–Kutta method; for step size h define

$$\begin{aligned} k_1 &= hf(t_i, y_i) \\ k_2 &= hf(t_i + h/2, y_i + k_1/2) \\ k_3 &= hf(t_i + h/2, y_i + k_2/2) \\ k_4 &= hf(t_i + h, y_i + k_3) \end{aligned}$$

   and evaluate the next value with

$$y_{i+1} = y_i + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

   Solve DE's above using this method. Compare exact and numerical solutions.