

Osnove matematične analize

Vaje, 12. teden

1. * Poišči minimum in maksimum funkcije $f(x, y) = xy$ na krogu

$$x^2 + y^2 \leq 1.$$

Rešitev: V notranjosti kroga ni ekstremov (točka $(0, 0)$ je sedlo). Na krožnici sta minimuma v $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ in $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, maksimuma sta v $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ in $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

2. Izračunaj naslednje nedoločene integrale:

(a) * $\int 3x^2 - 5x + 1 \, dx$,

(b) * $\int \frac{2+x}{1-x} \, dx$,

(c) $\int (1 + \frac{1}{x^2}) \sqrt{x}\sqrt{x} \, dx$,

(d) $\int \frac{3}{x-8} \, dx$,

(e) * $\int \frac{x}{(x-1)(x-8)} \, dx$,

(f) $\int \frac{1}{x^2-2x+2} \, dx$,

(g) * $\int \frac{1}{x(\log x)^2} \, dx$,

(h) $\int \tan x \, dx$,

(i) $\int \frac{e^x}{e^x-1} \, dx$,

(j) $\int x e^{-(x^2+1)} \, dx$,

(k) $\int \sin(3x) \, dx$,

(l) $\int \frac{\arcsin x}{\sqrt{1-x^2}} \, dx$,

(m) * $\int x^3 \log x \, dx$.

(n) $\int \frac{\log x}{x^2} \, dx$.

Rešitve: (a) $x^3 - \frac{5}{2}x^2 + x + c$, (b) $-x - 3 \log |1-x| + c$, (c) $\frac{4}{7}x^{\frac{7}{4}} - 4x^{-\frac{1}{4}} + c$,
(d) $3 \log |x-8| + c$, (e) $-\frac{1}{7} \log |x-1| + \frac{8}{7} \log |x-8| + c$, (f) $\arctan(x-1) + c$,
(g) $-\frac{1}{\log x} + c$, (h) $-\log |\cos(x)| + c$, (i) $\log |e^x - 1| + c$, (j) $-\frac{1}{2} e^{-(x^2+1)} + c$,
(k) $-\frac{1}{3} \cos(3x) + c$, (l) $\frac{(\arcsin x)^2}{2} + c$, (m) $\frac{1}{4} x^4 \log x - \frac{x^4}{16} + c$, (n) $-\frac{\log x}{x} - \frac{1}{x} + c$.

3. Izračunaj določene integrale

(a) * $\int_0^\pi x \sin(3x) \, dx$

(b) $\int_{-\pi}^\pi \cos(x) \sin^2(x) \, dx$

(c) * $\int_e^{e^2} \frac{(\log x)^2 - 2 \log x}{x} \, dx$

(d) * $\int_0^2 \frac{e^x}{e^{2x}+1} \, dx$

(e) $\int_0^{\sqrt{\log 2}} x e^{-x^2} \, dx$

(f) $\int_0^2 x e^{-x} \, dx$

(g) $\int_{-1}^2 \frac{x}{x^2-x-6} \, dx$

Rešitve: (a) $\frac{\pi}{3}$, (b) 0, (c) $-\frac{2}{3}$, (d) $\arctan(e^2) - \frac{\pi}{4}$, (e) $\frac{1}{4}$, (f) $1 - \frac{3}{e^2}$, (g) $-\frac{2 \log 2}{5}$.

4. Izračunaj ploščine likov, ki jih omejujejo dane krivulje

(a) * $y = x^2 + 2x$ in $y = x + 2$.

(b) $y = x^3 - x^2 + x$ in $y = 3x$.

(c) * $y^2 = 2x + 1$ in $y = x - 1$.

(d) $y = \sin(x)$ in $y = \cos(2x)$ na intervalu $[\frac{\pi}{6}, \frac{5\pi}{6}]$.

Rešitve: (a) $\frac{9}{2}$ (b) $\frac{37}{12}$ (c) $\frac{16}{3}$ (d) $\frac{3\sqrt{3}}{2}$