

Name: \_\_\_\_\_

Student ID number: \_\_\_\_\_

## Izpit iz Osnov matematične analize

### 1. september 2017

- You have: **45 minutes** to solve the problems
- Write your answers on this paper. Hand in also all your calculations and additional explanations.
- You are **not allowed** to use books, notes, communication devices of any kind or your neighbours.

#### 1. [20 points] Complex numbers

(a) What is the absolute value  $|z|$  of a complex number  $z$ ?

(b) Plot the sets  $A = \{z; |z - i| \leq 2\}$  and  $B = \{z; |z - i| < |z - 1|\}$  in the complex plane.

(c) Plot the images of the sets  $A$  and  $B$  by the map  $z \mapsto -1 + z(1 - i)$ .

(d) Finally, plot the images of  $A$  and  $B$  by the map  $z \mapsto z^2$ .

2. [20 points] Sequences and series

(a) The number  $L$  is the limit of the sequence  $(a_n)$  if \_\_\_\_\_  
\_\_\_\_\_.

(b) The number  $M$  is the upper limit of the sequence  $(b_n)$  if \_\_\_\_\_  
\_\_\_\_\_.

(c) The number  $M$  is the supremum of the sequence  $(b_n)$  if \_\_\_\_\_  
\_\_\_\_\_.

(d) You are given the sequence  $a_n = (-1)^n \left( \frac{n-1}{n+1} \right)$ .

- Is it bounded from above? If so, what is its supremum?
  
  
- Is it convergent? If yes, what is its limit, if no, why not?

3. [20 points] Functions

(a) The function  $f(x)$  is continuous at  $x_0$  if \_\_\_\_\_  
\_\_\_\_\_.

(b) For each of the following functions determine the value at 0 so that the function will be continuous at 0 or explain why such a value does not exist.

$$f(x) = |x|$$

$$f(x) = \frac{\sin x}{x}$$

$$f(x) = \frac{(\sin x)^2}{x}$$

$$f(x) = \frac{\sin x}{x^2}$$

$$f(x) = \arctan \frac{1}{x}$$

4. [20 points] Derivatives

(a) Write down the definition of the partial derivative  $f_x(x, y)$  of  $f$  at the point  $(a, b)$ .

(b) For the function  $f(x, y) = x^3 - 3xy - \frac{y}{x}$  write  $f_x(x, y)$ .

(c) Will the function value of the function given above increase or decrease after a small increase of the  $x$  coordinate at the point  $(1, 1)$ ?

(d) In which direction from the point  $(1, 1)$  will the function value of the function given above decrease the most?

5. [20 points] **Integral**

For the function  $F(x) = \int_0^x (1 - t^2)e^{-t} dt$

(a) write down its derivative,

(b) determine the domains of ascending and descending function values,

(c) determine the domains of convexity and concavity of the function,

(d) sketch the graph.