

# 1. popravni kolokvij iz Osnov matematične analize (Ljubljana, 26. januar 2017)

Čas reševanja: 90 minut. Naloge so enakovredne. Preberi celotno besedilo vsake naloge. Dovoljena je uporaba dveh listov velikosti A4 z obrazci. Rezultati bodo objavljeni na učilnica.fri.uni-lj.si.

**Justify your answers!**

1. Let  $w = \frac{4+2i}{3-i}$ .

- (a) Find the real and the imaginary part of  $w$ .
- (b) Write  $w$  in polar form.
- (c) Using the polar form and de Moivre formula find all solutions of the equation  $z^3 = w$  and draw them in the complex plane.

2. Function  $f$  is given by

$$f(x) = \begin{cases} ae^x & ; x \leq -2, \\ x + 1 & ; -2 < x < 1, \\ b + \frac{1}{x} & ; x \geq 1 \end{cases}$$

- (a) Find  $a$  and  $b$  such that  $f$  is continuous on the real axis.
- (b) Find the range of  $f$  (you can use the graph of the function).
- (c) Is  $f$  injective? Justify your answer!

3. For the function

$$f(x, y) = x^4 + 4xy + y^4 + 1$$

- (a) find and classify all stationary points,
- (b) compute the directional derivative at the point  $(1, 1)$  in the direction of  $\vec{a} = (2, 2)$ . Do the values of the function  $f$  increase or decrease in that direction?

4. (a) Compute the indefinite integral:

$$\int \frac{1}{e^x + 1} dx$$

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- (b) The function  $f : [0, \infty) \rightarrow \mathbb{R}$  is given by  $f(x) = \frac{\sqrt{2x}}{1+x^2}$ . Rotating the area between curve  $f$  and  $x$ -axis about the  $x$ -axis we obtain an unbounded solid  $D$ . Show that the solid  $D$  has finite volume and calculate it.

**Justify your answers!**