· Name and surname: ________________________ ID: ___________________

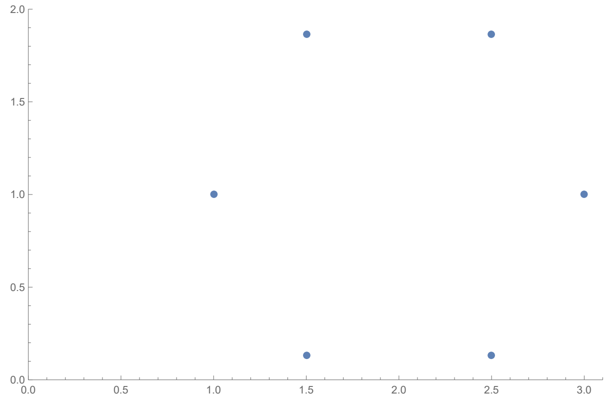
First exam for OMA, 19.01.2020

- Time limit: **30 minutes**
- For a passing grade you have to achieve at least 50% of all points. The number in the bracket $[\cdot]$ tells you how many points you get for a correct solution to the question.
- Any attempt of copying the solution of someone else, talking, using electronic equipment is **strictly** forbidden.

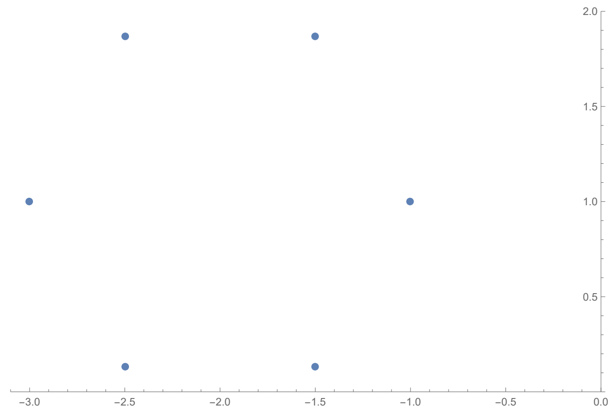
1. **[30 points]**

(a) Write down a rule for a transformation of the complex plane in the cartesion or the polar form, which maps the set $\{z \in \mathbb{C} \colon \operatorname{Re} z \geq 0, \operatorname{Im} z \geq 1\}$ onto the set $\{z \in \mathbb{C} \colon \operatorname{Re} z \leq 0, \operatorname{Im} z \geq 0\}$.

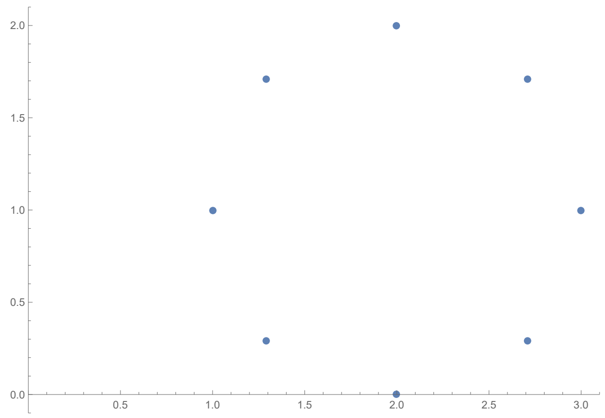
(b) Which of the following pictures represents the solutions of the equation $(z-2-i)^6 = 1$? Justify your answer.



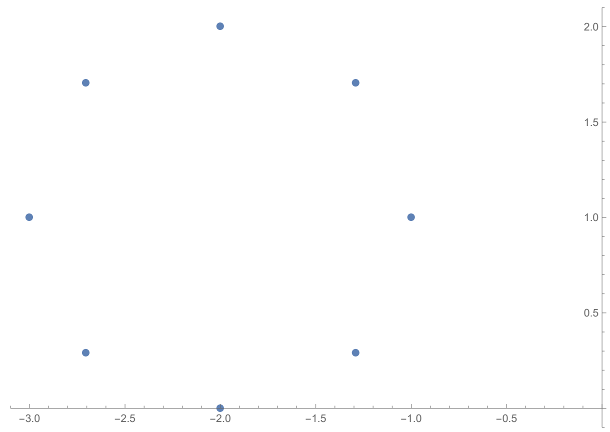
Slika 1



Slika 2



Slika 3



Slika 4

(c) We rotate the set of solutions of the equation in (1b) for the angle $\frac{\pi}{3}$ in a positive direction. Write down the equation, which is satisfied by the elements of the rotated set.

2. **[30 points]**

(a) Describe how you would determine the candidates for the extreme values of the continuously differentiable function $f : \mathcal{D}_f \to \mathbb{R}$ with the domain $\mathcal{D}_f = \{(x,y) \in \mathbb{R}^2 \colon x^2 + y^2 < 1\}$.

(b) We know that every continuous function $g : \mathcal{D}_g \to \mathbb{R}$, where $\mathcal{D}_g = \{(x,y) \in \mathbb{R}^2 \colon x^2 + y^2 \leq 1\}$, attains its global maximum and global minimum. How would you determine them if you know that g does not have stationary points on $\mathcal{D}_g$?

(c) Justify that every continuous function $f : [0,1] \to [0,1]$ has a fixed point, i.e., $\alpha \in [0,1]$ such that $f(\alpha) = \alpha$.

Hint: Define the function $g : [0,1] \to [0,1]$ with the rule $g(x) = f(x) - x$ and use the intermediate value theorem.

3. **[35 point]** Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is integrable on every interval $[a,b] \subset \mathbb{R}$, $a < b$.

(a) What is the domain of the function $F(x,y) = \displaystyle\int_y^x f(t)\, dt$?

(b) Let f be a strictly positive function. Determine the set of zeroes of the function F from (3a).

(c) Let $f(t) = t$. Describe the level curve of F, which contains the point $(2,0)$.

(d) Calculate both partial derivatives $\dfrac{\partial F}{\partial x}(x,y)$ and $\dfrac{\partial F}{\partial y}(x,y)$.