1. The system of equations

$$x - y + z - w = 1$$
$$x + y - z - w = 3$$

determines a two–dimensional plane in $\mathbb{R}^4$. Let $T(0,-1,-1,2)$. Our objective is to find the point on the plane which is closest to T.

(a) Write the matrix A and the right-hand side $\mathbf{b}$ of the system above.

(b) Evaluate A^+. (This is simple since A has full rank).

(c) Show that $P = I - A^+A$ is an orthogonal projection, meaning that $P^2 = P$ and $P^\mathsf{T} = P$. Onto which subspace does it project?

(d) Express and compute the solution using A^+.

(e) Write the function `pT = projekcija(A, b, T)`, which returns the projection of the point T on to the hyperplane defined by the system $A\mathbf{x} = \mathbf{b}$.

2. **Eigenvalues of (symmetric) matrices.** Suppose that $A \in \mathbb{R}^{n\times n}$ is a matrix with eigenvalues $\lambda_1,\ldots,\lambda_n$ such that $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$. Write an octave/Matlab function, which determines the largest eigenvalue with respect to absolute value and the corresponding eigenvector using the *power iteration*:

(0) Pick a nonzero $\mathbf{v} \in \mathbb{R}^n$.

(1) Evaluate $\mathbf{w} = A\mathbf{v}$.

(2) Evaluate $\mathbf{v} = \mathbf{w}/\|\mathbf{w}\|$ and repeat step (1).

Iteration is stopped once $\mathbf{v}$ is a good enough approximation to an eigenvector. The corresponding eigenvalue is then $\lambda = \mathbf{v}^\mathsf{T}A\mathbf{v}$. (Why?)

Assume now that $A \in \mathbb{R}^{n\times n}$ is a symmetric matrix with above properties. Adapt the power iteration to the *'simultaneous power iteration' (the QR iteration)* like this:

(0) Pick linearly independent $\mathbf{v}_1,\ldots,\mathbf{v}_m \in \mathbb{R}^n$, $m \leq n$, and assemble them into a matrix $V = [\mathbf{v}_1,\ldots,\mathbf{v}_m]$.

(0′) Determine the QR decomposition of V; $V = QR$

(1) Evaluate $W = AQ$.

(2) Determine the QR decomposition of W and repeat step (1).

The iteration is stopped once the columns of $Q = [\mathbf{q}_1,\ldots,\mathbf{q}_m]$ are good enough approximations to eigenvectors of A. The corresponding eigenvalues are again obtained from $\lambda_k = \mathbf{q}_k^\mathsf{T}A\mathbf{q}_k$. Why did we add the requirement that A is a symmetric matrix?

Test both methods and compare them with the built-in ones on some (not too big) test cases.