

## Assignment 1

Solve the following five exercises. Each exercise is worth three points. Solutions must be submitted by 13.3.2023. Use the link on e-ucilnica to turn in your work. The submission must be in pdf format.

### Exercise 1

a) Find the tight asymptotic bound of the algorithm given below.

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#### Algorithm 1

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```
1: function FUN(int  $n$ , int  $m$ )
2:   while  $n > 0$  do
3:      $m++$ ;
4:     for (int  $i = m$ ;  $i > 1$ ;  $i = i/4$ ) do
5:       UpdateData( $n, i$ ) // $\theta(n)$ 
6:     end for
7:      $m--$ 
8:     for (int  $i = 3m$ ;  $i > m$ ;  $i = i/2$ ) do
9:       UpdateDataFast( $n, 3m - i$ ) // $\theta(1)$ 
10:    end for
11:     $n--$ 
12:  end while
13:  return 0
14: end function
```

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b) Find the tight asymptotic bound of the following function  $f(a,s,e)$  in relation to  $e$ -s. Hint:  $a$  - array,  $s$  - start,  $e$  - end

```
int f(int[] a, int s, int e) {
    if (s == e)
        return 7;
    int p1 = 0;
    int j = e - s;

    for(int i = j; i > 1; i--)
        p1 += a[s + i] * a[e - i]

    int p2 = f(a, s, s + floor(j / 6));
    int p3 = f(a, s + floor(j / 6) + 1, e);
    return (p1+p2+p3);
}
```

c) Find the tight asymptotic bound of the following program in relation to variable N.

```

Move(N, source, dest, via){
  if(N == 0) then{
    MakeMove(N, source, dest) //O(1)
  }else{
    Move(N-1, source, via, dest);
    MakeMove(N, source, dest) //O(1)
    Move(N-1, via, dest, source);
  }
}

```

### Exercise 2

Solve the following two recurrences using the Masters method.

a)

$$T(n) = 4 \cdot T\left(\frac{n}{9}\right) + \sqrt[3]{n} \quad (1)$$

b)

$$T(n) = 5 \cdot T\left(\frac{n}{4}\right) + \log(n) \quad (2)$$

### Exercise 3

Solve the following two recurrences using the Akra-Bazi method.

a)

$$T(n) = 2T\left(\frac{n}{4}\right) + 3T\left(\frac{n}{6}\right) + \theta(n \log n) \quad (3)$$

b)

$$T(n) = T\left(\frac{n}{9}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{36}\right) + \sqrt{n^3} \quad (4)$$

### Exercise 4

a) Estimate the upper asymptotic bound of the given recurrence using the recursion-tree method.

$$T(n) = 8T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 27T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + n^3 \quad (5)$$

b) Prove the result you found in a) using the substitution method. (2 points)

d) Find the tight asymptotic bound of the given recurrence. (With any method)

### Exercise 5

a) Solve the following recurrence using the annihilator method. Find the exact solution. (2 points)

$$T(n) = T(n - 1) + n^2 + n - 1; T(0) = 1 \quad (6)$$

b) Solve the following recurrence using the annihilator method. Find the exact solution. (2 points)

$$\begin{aligned} T(n) &= 9T(n - 1) - 28T(n - 2) + 36T(n - 3) - 16T(n - 4) \\ T(0) &= 0; T(1) = 8 \\ T(2) &= 28; T(3) = 112 \end{aligned}$$