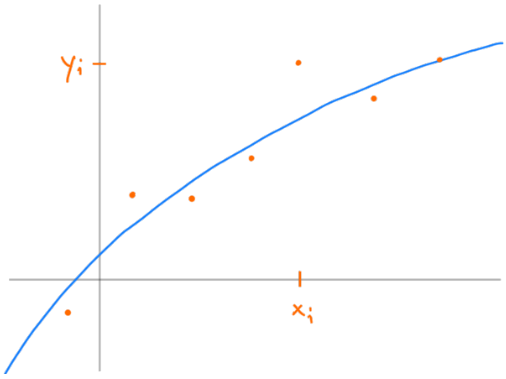
1. We'll only solve part (b) as it is far more practical. A naive way to think about the problem would be as follows: We're trying to fit the graph of

$$f(x) = \frac{ax}{b+x}$$



through the seven points (x_i, y_i) of our dataset. If those points were precisely on the graph of f, our task would be to solve the nonlinear system

$$f(x_i) = y_i \quad \text{or} \quad \frac{ax_i}{b+x_i} = y_i \quad \text{for} \quad i = 1, 2, \dots, 7.$$

Of course, those points do not lie on the graph of f, so we minimize the sum of squares of expressions

$$f(x_i) - y_i \quad \text{or} \quad \frac{ax_i}{b+x_i} - y_i \quad \text{for} \quad i = 1, 2, \dots, 7$$

instead. The sum of those squares is precisely the squared norm of

$$\vec{F}: \mathbb{R}^2 \to \mathbb{R}^7, \quad \vec{F}(\vec{x}) = \vec{F}\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} f(x_1) - y_1 \\ f(x_2) - y_2 \\ \vdots \\ f(x_7) - y_7 \end{bmatrix} = \begin{bmatrix} \frac{ax_1}{b+x_1} - y_1 \\ \vdots \\ \frac{ax_7}{b+x_7} - y_7 \end{bmatrix}.$$

The Jacobi matrix of $\vec{F}$ is:

$$J\vec{F}(\vec{x}) = J\vec{F}\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \frac{x_1}{b+x_1}, & -\frac{ax_1}{(b+x_1)^2} \\ \vdots & \vdots \\ \frac{x_7}{b+x_7}, & -\frac{ax_7}{(b+x_7)^2} \end{bmatrix}.$$

That's all we need to run the Gauss-Newton method.

2. Similarly as in the above exercise, instead of solving the system

$$(x-p_i)^2 + (y-q_i)^2 = d_i^2 \quad i = 1, 2, \dots, n,$$

we minimize the sum of squares of expressions

$$(x-p_i)^2 + (y-q_i)^2 - d_i^2 \quad i = 1, 2, \dots, n.$$

The corresponding vector-valued function is:

$$\vec{F}(\vec{x}) = \vec{F}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} (x-p_1)^2 + (y-q_1)^2 - d_1^2 \\ \vdots \\ (x-p_n)^2 + (y-q_n)^2 - d_n^2 \end{bmatrix}.$$

And its Jacobi matrix:

$$J\vec{F}(\vec{x}) = J\vec{F}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2(x-p_1), & 2(y-q_1) \\ \vdots & \vdots \\ 2(x-p_n), & 2(y-q_n) \end{bmatrix}.$$