



Digitalna vezja UL, FRI

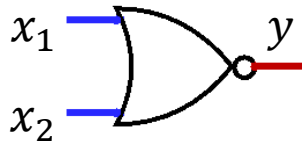


Vaja 2 Funkcije, PDNO, PKNO

Osnovne preklopne funkcije

Peircov op. (negirana disjunkcija) - NE ALI (NOR)

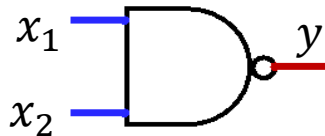
$$y = x_1 \downarrow x_2 = \overline{x_1 \vee x_2} = \overline{x_1} \cdot \overline{x_2}$$



x_1	x_2	$y = x_1 \downarrow x_2$
0	0	1
0	1	0
1	0	0
1	1	0

Shefferjev op. (negirana konjunkcija) - NE IN (NAND)

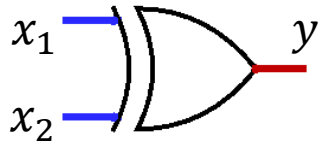
$$y = x_1 \uparrow x_2 = \overline{x_1 \cdot x_2} = \overline{x_1} \vee \overline{x_2}$$



x_1	x_2	$y = x_1 \uparrow x_2$
0	0	1
0	1	1
1	0	1
1	1	0

Ekskluzivni ALI (XOR)

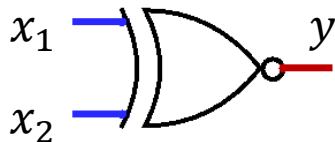
$$y = x_1 \nabla x_2 = \overline{x_1} \cdot x_2 \vee x_1 \cdot \overline{x_2}$$



x_1	x_2	$y = x_1 \nabla x_2$
0	0	0
0	1	1
1	0	1
1	1	0

Ekvivalenca (XNOR)

$$y = x_1 \equiv x_2 = \overline{x_1} \cdot \overline{x_2} \vee x_1 \cdot x_2$$



x_1	x_2	$y = x_1 \equiv x_2$
0	0	1
0	1	0
1	0	0
1	1	1

Implikacija

$$y = x_1 \rightarrow x_2 = \overline{x_1} \vee x_2$$

Komutativnost za implikacijo ne velja

$$x_1 \rightarrow x_2 \neq x_2 \rightarrow x_1$$

x_1	x_2	$y = x_1 \rightarrow x_2$
0	0	1
0	1	1
1	0	0
1	1	1

Poenostavitel preklonih funkcij

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 \uparrow x_2) \cdot \bar{x}_3 \\ &= (\overline{x_1 \cdot x_2}) \cdot \bar{x}_3 = (\bar{x}_1 \vee \bar{x}_2) \cdot \bar{x}_3 = \bar{x}_1 \cdot \bar{x}_3 \vee \bar{x}_2 \cdot \bar{x}_3\end{aligned}$$

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 \uparrow x_2) \cdot (x_2 \downarrow \bar{x}_3) \\ &= (\overline{x_1 \cdot x_2}) \cdot (\overline{x_2 \vee \bar{x}_3}) = (\bar{x}_1 \vee \bar{x}_2) \cdot (\bar{x}_2 \cdot x_3) = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3 \vee \bar{x}_2 \cdot \bar{x}_2 \cdot x_3 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3 \vee \bar{x}_2 \cdot x_3 \\ &= \bar{x}_2 \cdot x_3 (\bar{x}_1 \vee 1) = \bar{x}_2 \cdot x_3\end{aligned}$$

$$\begin{aligned}f(x_1, x_2) &= \overline{x_1 \equiv x_2} \\ &= \overline{\bar{x}_1 \cdot \bar{x}_2 \vee x_1 \cdot x_2} = (\overline{\bar{x}_1 \cdot \bar{x}_2}) \cdot (\overline{x_1 \cdot x_2}) = (x_1 \vee x_2) \cdot (\bar{x}_1 \vee \bar{x}_2) = x_1 \cdot \bar{x}_1 \vee x_1 \cdot \bar{x}_2 \vee \bar{x}_1 \cdot x_2 \vee \bar{x}_1 \cdot \bar{x}_1 \\ &= x_1 \cdot \bar{x}_2 \vee \bar{x}_1 \cdot x_2 = x_1 \nabla x_2\end{aligned}$$

$$\begin{aligned}f(x_1, x_2, x_3) &= ((x_1 \uparrow x_2) \uparrow x_3) \cdot (\bar{x}_2 \vee x_3) \\ &= (\overline{(x_1 \uparrow x_2) \cdot x_3}) \cdot (\bar{x}_2 \vee x_3) = (\overline{(x_1 \uparrow x_2) \vee \bar{x}_3}) \cdot (\bar{x}_2 \vee x_3) \\ &= (\overline{(\overline{\bar{x}_1 \cdot \bar{x}_2}) \vee \bar{x}_3}) \cdot (\bar{x}_2 \vee x_3) = (x_1 \cdot x_2 \vee \bar{x}_3) \cdot (\bar{x}_2 \vee x_3) = x_1 \cdot x_2 \cdot \bar{x}_2 \vee x_1 \cdot x_2 \cdot x_3 \vee \bar{x}_2 \cdot \bar{x}_3 \vee x_3 \cdot \bar{x}_3 \\ &= x_1 \cdot x_2 \cdot x_3 \vee \bar{x}_2 \cdot \bar{x}_3\end{aligned}$$

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 \rightarrow x_3) \cdot (x_2 \vee \bar{x}_3) \\ &= (\bar{x}_1 \vee x_3) \cdot (x_2 \vee \bar{x}_3) = \bar{x}_1 \cdot x_2 \vee \bar{x}_1 \cdot \bar{x}_3 \vee x_2 \cdot x_3 \vee x_3 \cdot \bar{x}_3 = \bar{x}_1 \cdot x_2 \vee \bar{x}_1 \cdot \bar{x}_3 \vee x_2 \cdot x_3\end{aligned}$$



Zapis popolnih oblik preklopnih funkcij

Normalna oblika – dva nivoja operatorjev

Popolna oblika – v izrazih prvega nivoja so vse spremenljivke

PDNO – popolna disjunktivna normalna oblika

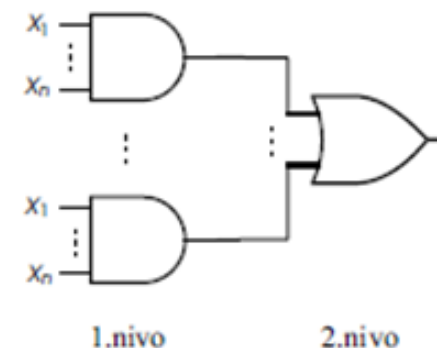
$$f(x_1, x_2, \dots, x_n) = \bigvee_{i=0}^{2^n-1} m_i f(\vec{w}_i)$$

$f(\vec{w}_i)$... vrednost funkcije pri i -tem vhodnem vektorju (vrstici)

m_i ... minterm i ; $m_i = x_1^{w_{1,i}} \cdot x_2^{w_{2,i}} \cdot \dots \cdot x_n^{w_{n,i}}$; $i = 0, 1, 2, \dots, 2^n - 1$

$$x^w = \begin{cases} x, & w = 1 \\ \bar{x}, & w = 0 \end{cases}$$

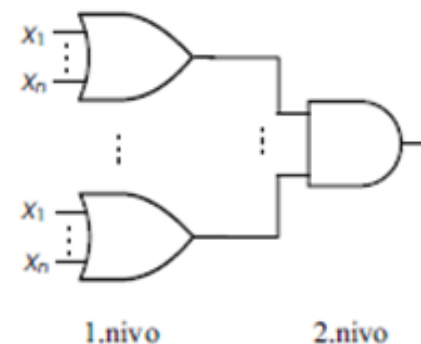
$w_{j,i}$... j -ti bit binarnega zapisa števila i



PKNO – popolna konjunktivna normalna oblika

$$f(x_1, x_2, \dots, x_n) = \&_{i=0}^{2^n-1} (M_{2^n-1-i} \vee f(\vec{w}_i))$$

M_{2^n-1-i} ... maksterm $2^n - 1 - i$; $M_{2^n-1-i} = \bar{x}_1^{\bar{w}_{1,i}} \vee \bar{x}_2^{\bar{w}_{2,i}} \vee \dots \vee \bar{x}_n^{\bar{w}_{n,i}}$; $i = 0, 1, 2, \dots, 2^n - 1$



Primer: Preklopna funkcija $f(x_1, x_2, x_3)$

Minterm – konjunkcija vhodnih spremenljivk x_1, x_2, x_3 . Spremenljivka x_i je negirana, če je vhodna vrednost enaka 0, in je nenegirana, če je vhodna vrednost enaka 1.

Maksterm - disjunkcija vhodnih spremenljivk x_1, x_2, x_3 . Spremenljivka x_i je negirana, če je vhodna vrednost enaka 1 in je nenegirana, če je vhodna vrednost enaka 0.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm	maksterm
0	0	0	$f(\vec{w}_0)$	$m_0 = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_7 = x_1 \vee x_2 \vee x_3$
0	0	1	$f(\vec{w}_1)$	$m_1 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3$	$M_6 = x_1 \vee x_2 \vee \bar{x}_3$
0	1	0	$f(\vec{w}_2)$	$m_2 = \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$	$M_5 = x_1 \vee \bar{x}_2 \vee x_3$
0	1	1	$f(\vec{w}_3)$	$m_3 = \bar{x}_1 \cdot x_2 \cdot x_3$	$M_4 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$
1	0	0	$f(\vec{w}_4)$	$m_4 = x_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_3 = \bar{x}_1 \vee x_2 \vee x_3$
1	0	1	$f(\vec{w}_5)$	$m_5 = x_1 \cdot \bar{x}_2 \cdot x_3$	$M_2 = \bar{x}_1 \vee x_2 \vee \bar{x}_3$
1	1	0	$f(\vec{w}_6)$	$m_6 = x_1 \cdot x_2 \cdot \bar{x}_3$	$M_1 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$
1	1	1	$f(\vec{w}_7)$	$m_7 = x_1 \cdot x_2 \cdot x_3$	$M_0 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

PDNO

Zapišite PDNO za preklopno funkcijo $f(x_1, x_2, x_3) = x_1 \cdot \bar{x}_2 \vee x_2 \cdot x_3$.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm
0	0	0	0	$m_0 = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$
0	0	1	0	$m_1 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3$
0	1	0	0	$m_2 = \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$
0	1	1	1	$m_3 = \bar{x}_1 \cdot x_2 \cdot x_3$
1	0	0	1	$m_4 = x_1 \cdot \bar{x}_2 \cdot \bar{x}_3$
1	0	1	1	$m_5 = x_1 \cdot \bar{x}_2 \cdot x_3$
1	1	0	0	$m_6 = x_1 \cdot x_2 \cdot \bar{x}_3$
1	1	1	1	$m_7 = x_1 \cdot x_2 \cdot x_3$

$$\begin{aligned} f(x_1, x_2, x_3) &= m_0 \cdot 0 \vee m_1 \cdot 0 \vee m_2 \cdot 0 \vee m_3 \cdot 1 \vee m_4 \cdot 1 \vee m_5 \cdot 1 \vee m_6 \cdot 0 \vee m_7 \cdot 1 \\ &= m_3 \vee m_4 \vee m_5 \vee m_7 = \\ &= \vee^3 (3, 4, 5, 7) = \\ &= \bar{x}_1 \cdot x_2 \cdot x_3 \vee x_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \vee x_1 \cdot \bar{x}_2 \cdot x_3 \vee x_1 \cdot x_2 \cdot x_3 \end{aligned}$$

PKNO

Zapište PKNO za preklapno funkciju $f(x_1, x_2, x_3) = x_1 \cdot \bar{x}_2 \vee x_2 \cdot x_3$.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	maksterm
0	0	0	0	$M_7 = x_1 \vee x_2 \vee x_3$
0	0	1	0	$M_6 = x_1 \vee x_2 \vee \bar{x}_3$
0	1	0	0	$M_5 = x_1 \vee \bar{x}_2 \vee x_3$
0	1	1	1	$M_4 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$
1	0	0	1	$M_3 = \bar{x}_1 \vee x_2 \vee x_3$
1	0	1	1	$M_2 = \bar{x}_1 \vee x_2 \vee \bar{x}_3$
1	1	0	0	$M_1 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$
1	1	1	1	$M_0 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$$\begin{aligned} f(x_1, x_2, x_3) &= (M_7 \vee 0) \cdot (M_6 \vee 0) \cdot (M_5 \vee 0) \cdot (M_4 \vee 1) \cdot (M_3 \vee 1) \cdot (M_2 \vee 1) \cdot (M_1 \vee 0) \cdot (M_0 \vee 1) \\ &= M_7 \cdot M_6 \cdot M_5 \cdot M_1 = \\ &= \&^3(7, 6, 5, 1) = \\ &= (x_1 \vee x_2 \vee x_3) \cdot (x_1 \vee x_2 \vee \bar{x}_3) \cdot (x_1 \vee \bar{x}_2 \vee x_3) \cdot (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \end{aligned}$$

Naloga

Zapišite PDNO in PKNO za preklopno funkcijo $f(x_1, x_2, x_3)$, ki zavzame vrednost 1, če sta dve vhodni spremenljivki enaki 0 ali tri vhodne spremenljivke enake 1.

x_1	x_1	x_1	$f(x_1, x_2, x_3)$	minterm	maksterm
0	0	0		$m_0 = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_7 = x_1 \vee x_2 \vee x_3$
0	0	1		$m_1 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3$	$M_6 = x_1 \vee x_2 \vee \bar{x}_3$
0	1	0		$m_2 = \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$	$M_5 = x_1 \vee \bar{x}_2 \vee x_3$
0	1	1		$m_3 = \bar{x}_1 \cdot x_2 \cdot x_3$	$M_4 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$
1	0	0		$m_4 = x_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_3 = \bar{x}_1 \vee x_2 \vee x_3$
1	0	1		$m_5 = x_1 \cdot \bar{x}_2 \cdot x_3$	$M_2 = \bar{x}_1 \vee x_2 \vee \bar{x}_3$
1	1	0		$m_6 = x_1 \cdot x_2 \cdot \bar{x}_3$	$M_1 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$
1	1	1		$m_7 = x_1 \cdot x_2 \cdot x_3$	$M_0 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

PDNO: $f(x_1, x_2, x_3) = ?$

PKNO: $f(x_1, x_2, x_3) = ?$

Rešitev

Zapišite PDNO in PKNO za preklopno funkcijo $f(x_1, x_2, x_3)$, ki zavzame vrednost 1, če sta dve vhodni spremenljivki enaki 0 ali tri vhodne spremenljivke enake 1.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$	minterm	maksterm
0	0	0	0	$m_0 = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_7 = x_1 \vee x_2 \vee x_3$
0	0	1	1	$m_1 = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3$	$M_6 = x_1 \vee x_2 \vee \bar{x}_3$
0	1	0	1	$m_2 = \bar{x}_1 \cdot x_2 \cdot \bar{x}_3$	$M_5 = x_1 \vee \bar{x}_2 \vee x_3$
0	1	1	0	$m_3 = \bar{x}_1 \cdot x_2 \cdot x_3$	$M_4 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$
1	0	0	1	$m_4 = x_1 \cdot \bar{x}_2 \cdot \bar{x}_3$	$M_3 = \bar{x}_1 \vee x_2 \vee x_3$
1	0	1	0	$m_5 = x_1 \cdot \bar{x}_2 \cdot x_3$	$M_2 = \bar{x}_1 \vee x_2 \vee \bar{x}_3$
1	1	0	0	$m_6 = x_1 \cdot x_2 \cdot \bar{x}_3$	$M_1 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$
1	1	1	1	$m_7 = x_1 \cdot x_2 \cdot x_3$	$M_0 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$$\begin{aligned} \text{PDNO: } f(x_1, x_2, x_3) &= m_1 \vee m_2 \vee m_4 \vee m_7 = \vee^3 (1, 2, 4, 7) \\ &= \bar{x}_1 \cdot \bar{x}_2 \cdot x_3 \vee \bar{x}_1 \cdot x_2 \cdot \bar{x}_3 \vee x_1 \cdot \bar{x}_2 \cdot \bar{x}_3 \vee x_1 \cdot x_2 \cdot x_3 \end{aligned}$$

$$\begin{aligned} \text{PKNO: } f(x_1, x_2, x_3) &= M_7 \cdot M_4 \cdot M_2 \cdot M_1 = \&^3 (7, 4, 2, 1) \\ &= (x_1 \vee x_2 \vee x_3) \cdot (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \cdot (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \cdot (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \end{aligned}$$