1) $|\psi\rangle$ element Hilbertovega prostora, $\langle\psi|\psi\rangle = 1$

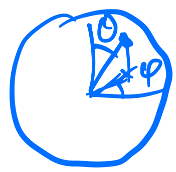
2) opazljivka $\longleftrightarrow$ $\hat{A} = \hat{A}^{\dagger}$

3) Meritev: rezultat λ_i z verjetnostjo $p_i = |\langle v_i|\psi\rangle|^2$

4) Kolaps valovne funkcije: $|\psi'\rangle = |v_i\rangle$

5) Časovni potek: $|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$ $\quad U^{\dagger}U = UU^{\dagger} = 1$

$$|\psi\rangle = \cos\theta/2\,|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}\,|1\rangle$$



1) Tenzorski produkt

$$V\otimes W = \{|v\rangle\otimes|w\rangle\} \qquad \underline{|i\rangle\otimes|j\rangle} = |i\rangle|j\rangle = |i,j\rangle = |ij\rangle$$

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

Bilinearnost: $(\alpha|0\rangle+\beta|1\rangle)\otimes(\gamma|0\rangle+\delta|1\rangle) = \alpha\gamma|00\rangle+\alpha\delta|01\rangle+\beta\gamma|10\rangle+\beta\delta|11\rangle$

$$= \begin{pmatrix}\alpha\gamma\\ \alpha\delta\\ \beta\gamma\\ \beta\delta\end{pmatrix}$$

$$(\hat{A}\otimes\hat{B})(|v\rangle\otimes|w\rangle) = (\hat{A}|v\rangle)\otimes(\hat{B}|w\rangle)$$

Kroneckerjev produkt:

$$\underset{m\times n}{A}\otimes\underset{p\times q}{B} = \begin{bmatrix} A_{11}B & A_{12}B & & A_{1n}B\\ A_{21}B & \ddots & & \\ A_{m1}B & & \dots & A_{mn}B\end{bmatrix}$$

$mp\times nq$

$$\underset{\begin{bmatrix}1&0\\0&1\end{bmatrix}}{\underset{\uparrow}{1\!\!1}}\otimes\underset{\begin{bmatrix}1&0\\0&1\end{bmatrix}}{\underset{\uparrow}{1\!\!1}} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \qquad \underset{\begin{bmatrix}1&0\\0&1\end{bmatrix}}{1\!\!1}\otimes\underset{\begin{bmatrix}0&1\\1&0\end{bmatrix}}{X} = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \qquad \underset{\begin{bmatrix}0&1\\1&0\end{bmatrix}}{\underset{\uparrow}{X}}\otimes 1\!\!1 = \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}$$

$$1\!\!1\otimes X \neq X\otimes 1\!\!1$$

D.N. $X \otimes Z$, $Z \otimes X$

Potence: $|\psi\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle)$

$$|\psi\rangle \otimes |\psi\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle) \otimes 1/\sqrt{2}\,(|0\rangle + |1\rangle)$$
$$= 1/2\,(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle = \dots = |\psi\rangle^{\otimes 3}$$

$$(|v_1\rangle \otimes |w_1\rangle, |v_2\rangle \otimes |w_2\rangle) = (\langle v_1| \otimes \langle w_1|)(|w_2\rangle \otimes |w_2\rangle)$$
$$= \langle v_1|v_2\rangle \langle w_1|w_2\rangle$$

2) Več kubitov

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

8 realnih parametrov

$$\langle\psi|\psi\rangle = 1 \quad \Rightarrow \quad |\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

α_{00} realen

$\Rightarrow$ 6 prostih parametrov

3) Prepoved kloniranja

Wooters, Zurek (1982), Dieks (1982)

$$U(|\phi\rangle|e\rangle) = |\phi\rangle|\phi\rangle \to \langle\phi|\langle e|U^\dagger = \langle\phi|\langle\phi| \quad U^\dagger U = 1$$

- $U(|\psi\rangle|e\rangle) = |\psi\rangle|\psi\rangle$

$$\langle\phi|\psi\rangle = \langle\phi|\psi\rangle\overset{1}{\langle e|e\rangle} = (\langle\phi|\langle e|)(|\psi\rangle|e\rangle)$$
$$= \underbrace{\langle\phi|\langle e|U^\dagger}\underbrace{U|\psi\rangle|e\rangle}$$
$$= \langle\phi|\langle\phi|\cdot|\psi\rangle|\psi\rangle = \langle\phi|\psi\rangle\langle\phi|\psi\rangle = \langle\phi|\psi\rangle^2$$

$a^2 = a \quad$ Ali $a = 0$ ali $a = 1$.

$\langle \phi | \psi \rangle = 0$ $\quad \phi$ in ψ ortogonalna $\quad P_\phi = |\phi\rangle\langle\phi| \quad P_\psi = |\psi\rangle\langle\psi|$

$\hat{A} = 1 P_\phi - 1 P_\psi$

$\langle \phi | \psi \rangle = 1$ $\quad \phi$ in ψ isto stanje.

4. Meritve v večdelčnih sistemih

$|\psi\rangle = 1/2\,|00\rangle - i/2\,|\underline{10}\rangle + 1/\sqrt{2}\,|\underline{11}\rangle$

a)

Rezultat	Verjetnost		Končno stanje
00	$|1/2|^2 = 1/4$	25%	$|\psi'\rangle = |00\rangle$
01	0		/
10	$|-i/2|^2 = 1/4$	25%	$|\psi'\rangle = |10\rangle$
11	$|1/\sqrt{2}|^2 = 1/2$	50%	$|\psi'\rangle = |11\rangle$

b) Ali je prvi enak 1?

$P(A \cup B) = P(A) + P(B) - P(\cancel{A \cap B})^{\,0}$

$|-i/2|^2 + |1/\sqrt{2}|^2 = 3/4 \leftarrow$

Končno stanje: $\quad |\tilde{\psi'}\rangle = -i/2\,|10\rangle + 1/\sqrt{2}\,|11\rangle$

$|\psi'\rangle = -\frac{i}{\sqrt{3}}\,|10\rangle + \frac{\sqrt{2}}{\sqrt{3}}\,|11\rangle$ ✓ $\quad |-i/\sqrt{3}|^2 + \left|\sqrt{\frac{2}{3}}\right|^2 = \frac{1}{3} + \frac{2}{3} = 1$

Drugi enak 1?

$\left|\sqrt{\frac{2}{3}}\right|^2 = 2/3 \qquad 66{,}6\%$

Verjetnost, da je bilo stanje 11:

$\underbrace{p(A \cap B)}_{|11\rangle} = p(A)\,p(B|A) = 3/4 \cdot 2/3 = 1/2$ Konsistentno!

c) Drugi enak 1?

$|1/\sqrt{2}|^2 = 1/2 \rightarrow 50\% \qquad |\psi'\rangle = |11\rangle$

Prvi meri 1? → 100% $p(A\cap B) = p(B)\,p(A|B) = 1/2 \cdot 1 = 1/2$ ✓

Rezultati neodvisni od kraja in časa meritev.

5) **Bellova stanja**

$|\psi\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$

Naš laboratorij		Po meritvi		V vesolju
0	→	$\|\psi'\rangle = \|00\rangle$	→	0
1	→	$\|\psi'\rangle = \|11\rangle$	→	1

} rezultat meritev korelirano

Einstein, Podolski, Rosen (1935)

Bellova stanja:

$|\beta_{00}\rangle = |\Phi^+\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$ $|\beta_{01}\rangle = |\Psi^+\rangle = 1/\sqrt{2}\,(|01\rangle + |10\rangle)$

$|\beta_{10}\rangle = |\Phi^-\rangle = 1/\sqrt{2}\,(|00\rangle - |11\rangle)$ $|\beta_{11}\rangle = |\Psi^-\rangle = 1/\sqrt{2}\,(|01\rangle - |10\rangle)$

popolnoma korelirano popolnoma antikorelirano

$|+\rangle, |-\rangle$ $|+\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle)$ $|-\rangle = 1/\sqrt{2}\,(|0\rangle - |1\rangle)$

$|00\rangle + |11\rangle = |++\rangle + |--\rangle$

$$|++\rangle + |--\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle) \otimes 1/\sqrt{2}\,(|0\rangle + |1\rangle) + 1/\sqrt{2}\,(|0\rangle - |1\rangle) \otimes 1/\sqrt{2}\,(|0\rangle - |1\rangle)$$

$$= 1/2\,(|00\rangle + \cancel{|01\rangle} + \cancel{|10\rangle} + |11\rangle) + 1/2\,(|00\rangle - \cancel{|01\rangle} - \cancel{|10\rangle} + |11\rangle)$$

$$= |00\rangle + |11\rangle$$

Maksimalno prepletena. $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

$$|00\rangle + |01\rangle + |10\rangle + |11\rangle = (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$|\psi_1\rangle = |0\rangle + |1\rangle$
$|\psi_2\rangle = |0\rangle + |1\rangle$

$$= |\psi_1\rangle \otimes |\psi_2\rangle$$

Separabilna stanja.

6) No-communication (no-signaling)

Herbert (1982) FLASH

$|\psi\rangle = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$ Alice Bob

0 A. opravi meritev v bazi $|0\rangle, |1\rangle$ Bob $|\psi'\rangle = |0\rangle$ ali $|1\rangle$

1 A. opravi meritev v bazi $|+\rangle, |-\rangle$ $|\psi'\rangle = |+\rangle$ ali $|-\rangle$

0	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	100% 0 ali 100% 1
1	$\|+\rangle$	$\|+\rangle$	$\|+\rangle$	$\|-\rangle$	$\|-\rangle$	$\|-\rangle$	50% 0 in 50% 1

} ločljivo

Ne deluje!

7) Kvantna vezja

— kubit

—☐— operacije (unitarne)

=☐=

→ ni razcepa

= klasični bit

meritev $|0\rangle \to 0$ $|1\rangle \to 1$ $\alpha|0\rangle + \beta|1\rangle \to p_0 = |\alpha|^2$ $p_1 = |\beta|^2$

Čas: z leve proti desni. $\hat{A}\hat{B}\hat{C}|\psi\rangle$, $\hat{C}$ potem $\hat{B}$ potem $\hat{A}$

ni zank

8) Kvantna vrata

$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ $|\psi\rangle \to |\psi'\rangle$

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$ $X|1\rangle = |0\rangle$

Vrata NOT, NE, bit-flip. $X^2 = \mathbb{1}$

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$ $Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Vrata phase-flip. $Z^2 = \mathbb{1}$

Hadamardova vrata H: $H|0\rangle = |+\rangle$ $H|1\rangle = |-\rangle$

$H = 1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $H^2 = 1/2\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = 1/2\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \mathbb{1}$ ✓ involucija

$H^\dagger = H^{-1} = H$ $H|+\rangle = |0\rangle$ $H|-\rangle = |1\rangle$

$H \otimes H\ |00\rangle \rightarrow |++\rangle = 1/2\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

$H^{\otimes n}\ |0\rangle^{\otimes n} \rightarrow |+\rangle^{\otimes n}$

Relacija duli osi $\hat{x} + \hat{z}$ za $180°$ (π). D.N. $HXH = Z$

$HZH = X$

phase-shift $P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}$ $P(\varphi)|0\rangle = |0\rangle$ $P(\varphi)|1\rangle = e^{i\varphi}|1\rangle$

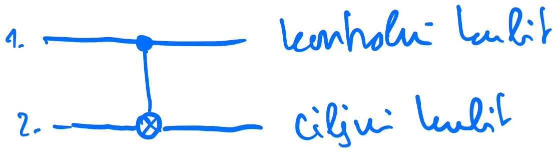
$Z = P(\pi)$ $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = P(\pi/2) = \sqrt{Z}$

$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} = P(\pi/4) = \sqrt{S} = \sqrt[4]{Z}$

$\pi/8$ vrata

3) Večkubitna vrata

Pogojena vrata, kontrolni vhod. Če enak $|0\rangle$, ciljnega ne spremenimo
če enak $|1\rangle$, delujemo z neko operacijo U.

CNOT 

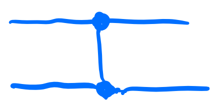
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} \psi_{1,1}|\psi_2\rangle \\ \psi_{1,2}|\psi_2\rangle \end{bmatrix} = \begin{bmatrix} \psi_{1,1}\,\psi_{2,1} \\ \psi_{1,1}\,\psi_{2,2} \\ \psi_{1,2}\,\psi_{2,1} \\ \psi_{1,2}\,\psi_{2,2} \end{bmatrix}$$

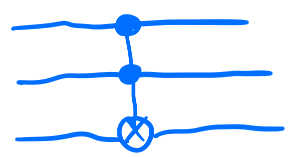
$$U_{CNOT} = \begin{bmatrix} \boxed{\mathbb{1}} & \emptyset \\ \emptyset & \boxed{X} \end{bmatrix} \qquad |A,B\rangle \xrightarrow{U} |A, A\oplus B\rangle$$

$A,B \in \{0,1\}$

$\uparrow$ vsota modulo 2, XOR

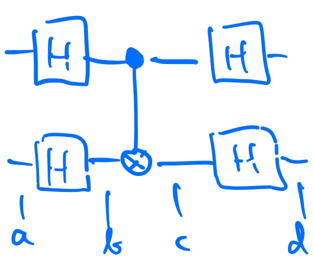
$$\left.\begin{aligned} |00\rangle &\rightarrow |00\rangle \\ |01\rangle &\rightarrow |01\rangle \\ |10\rangle &\rightarrow |11\rangle \\ |11\rangle &\rightarrow |10\rangle \end{aligned}\right\} \text{flip!} \qquad U^2 = \mathbb{1}$$

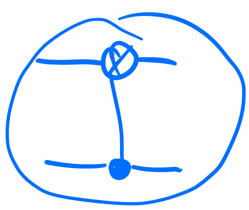
CZ: 

Toffoli: (CCNOT) 

$$|A,B,C\rangle \rightarrow |A,B,C\oplus AB\rangle$$

Univerzalni set: CNOT, vsa enokubitna.



$\equiv$ 

$$\overset{a}{|00\rangle} \rightarrow \overset{b}{\tfrac{1}{2}}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) \rightarrow \overset{c}{\tfrac{1}{2}}\left(|00\rangle + |01\rangle + |11\rangle + |10\rangle\right) = \overset{d}{|00\rangle}$$

$(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$

$$|01\rangle \rightarrow \tfrac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right) \rightarrow \tfrac{1}{2}\left(|00\rangle - |01\rangle + |11\rangle - |10\rangle\right)$$

$$= \tfrac{1}{\sqrt{2}}\left(\overset{1\to}{|0\rangle - |1\rangle}\right) \otimes \tfrac{1}{\sqrt{2}}\left(\overset{1\to}{|0\rangle - |1\rangle}\right) \rightarrow |11\rangle$$

$(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)$

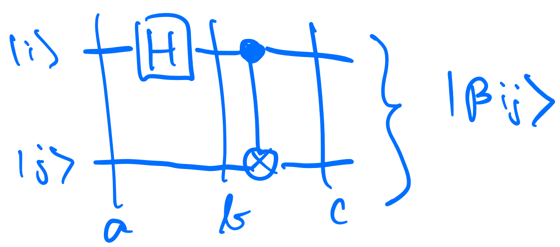
$$|10\rangle \rightarrow \tfrac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right) \rightarrow \tfrac{1}{2}\left(|00\rangle + |01\rangle - |11\rangle - |10\rangle\right)$$

$= 1/\sqrt{2}(|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to |10\rangle$

$\otimes\ |11\rangle \longrightarrow 1/2\,(|00\rangle - |01\rangle - |10\rangle + |11\rangle) \longrightarrow 1/2\,(|00\rangle - |01\rangle - |11\rangle + |10\rangle)$

$=$ $\longrightarrow |01\rangle$ ✓

10) <u>Entangler</u>



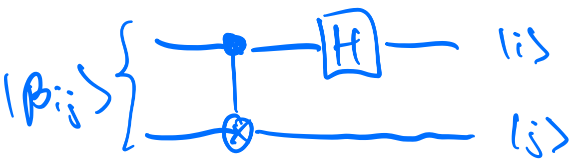
$$\begin{array}{lllll}
ij & & b & & c \\
|00\rangle & \longrightarrow & 1/\sqrt{2}(|0\rangle + |1\rangle) \otimes |0\rangle = 1/\sqrt{2}(|00\rangle + |10\rangle) & \longrightarrow & \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \checkmark \\
|01\rangle & \longrightarrow & 1/\sqrt{2}(|0\rangle + |1\rangle) \otimes |1\rangle = 1/\sqrt{2}(|01\rangle + |11\rangle) & \longrightarrow & 1/\sqrt{2}(|01\rangle + |10\rangle) \\
 & & & & |\Psi^+\rangle = |\beta_{01}\rangle \\
|10\rangle & \longrightarrow & 1/\sqrt{2}(|0\rangle - |1\rangle) \otimes |0\rangle = 1/\sqrt{2}(|00\rangle - |10\rangle) & \longrightarrow & 1/\sqrt{2}(|00\rangle - |11\rangle) \\
 & & & & |\beta_{10}\rangle = |\Phi^-\rangle \\
|11\rangle & \longrightarrow & 1/\sqrt{2}(|0\rangle - |1\rangle) \otimes |1\rangle = 1/\sqrt{2}(|01\rangle - |11\rangle) & \longrightarrow & 1/\sqrt{2}(|01\rangle - |10\rangle) \\
 & & & & |\beta_{11}\rangle = |\Psi^-\rangle
\end{array}$$

$$|\beta_{ij}\rangle = \frac{|0,j\rangle + (-1)^i |1,\bar{j}\rangle}{\sqrt{2}} \qquad \bar{j} \to \quad \bar{0} = 1 \quad \bar{1} = 0$$

$$|\Phi^+\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$$

$$(X \otimes 1)|\Phi^+\rangle = |\Psi^+\rangle \qquad (1 \otimes Z)|\Phi^+\rangle = |\Phi^-\rangle$$

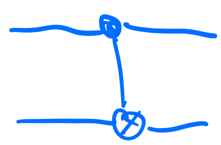
$$(1 \otimes X)|\Phi^+\rangle = |\Psi^+\rangle \qquad (1 \otimes XZ)|\Phi^+\rangle = |\Psi^-\rangle$$



11) <u>Kopiranje</u>

$\alpha|00\rangle + \beta|10\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$|0\rangle$

$|\psi'\rangle \; \alpha|00\rangle + \beta|11\rangle$

$$|\psi'\rangle = |\psi\rangle \otimes |\psi\rangle \; ? \; = (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

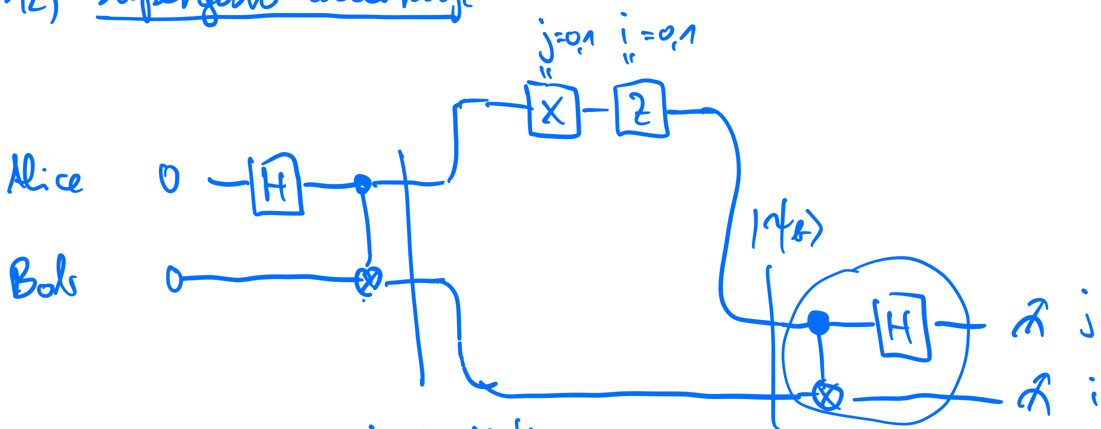
$$= \alpha^2|00\rangle + \alpha\beta(|01\rangle + |10\rangle) + \beta^2|11\rangle$$

$$= \alpha|00\rangle + \beta|11\rangle$$

$\alpha^2 = \alpha$

$\beta^2 = \beta \qquad \alpha\beta = 0 \longrightarrow \begin{cases} \alpha = 1 & \beta = 0 \\ \alpha = 0 & \beta = 1 \end{cases} \qquad |\psi\rangle = |0\rangle$ ali $|1\rangle$

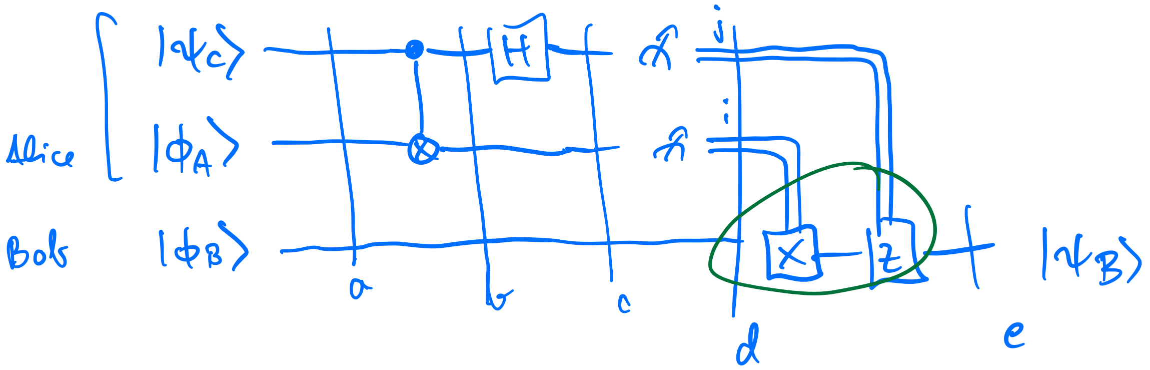
12) <u>Supergosto kodiranje</u>



$|\beta_{00}\rangle = |\Phi^+\rangle$

i	j	U_{Alice}	$\lvert\psi_B\rangle$		$i\;j$
0	0	$\mathbb{1}$	$\lvert\beta_{00}\rangle = \lvert\Phi^+\rangle$	$\rightarrow$	00
0	1	X	$\lvert\Psi^+\rangle$	$\rightarrow$	01
1	0	Z	$\lvert\Phi^-\rangle$	$\rightarrow$	10
1	1	ZX	$\lvert\Psi^-\rangle$	$\rightarrow$	11

13) Kvantna teleportacija : prenos stanja kubita na drugo mesto



$|\psi_B\rangle = |\psi_C\rangle$

$|\phi_{AB}\rangle = 1/\sqrt{2}\left(|00\rangle + |11\rangle\right)$

$|\psi_C\rangle = \alpha|0\rangle + \beta|1\rangle$

$$|\overset{i}{\phi_A}\overset{j}{\phi_B}\psi_C\rangle = \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|001\rangle + \frac{\alpha}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|111\rangle$$

$$\xrightarrow{b} \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|101\rangle + \frac{\alpha}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|011\rangle$$

$$\xrightarrow{c} \frac{\alpha}{2}\left(|000\rangle + |001\rangle\right) + \frac{\beta}{2}\left(|100\rangle - |101\rangle\right)$$
$$+ \frac{\alpha}{2}\left(|110\rangle + |111\rangle\right) + \frac{\beta}{2}\left(|010\rangle - |011\rangle\right)$$

$i\ j$	$\lvert\phi_B^{(d)}\rangle$	končno stanje
0 0	$\alpha\lvert 0\rangle + \beta\lvert 1\rangle = \lvert\psi_C\rangle$	$\lvert\psi_C\rangle$
0 1	$\alpha\lvert 0\rangle - \beta\lvert 1\rangle = Z\lvert\psi_C\rangle$	$\lvert\psi_C\rangle$
1 0	$\beta\lvert 0\rangle + \alpha\lvert 1\rangle = X\lvert\psi_C\rangle$	$\lvert\psi_C\rangle$
1 1	$-\beta\lvert 0\rangle + \alpha\lvert 1\rangle = XZ\lvert\psi_C\rangle$	$\lvert\psi_C\rangle$