Rešitve 2. kolokvija (12.1.2023)

1. naloga (25 točk)

Dana je funkcija f s predpisom $f(x) = \frac{x}{e^{1-x}}$.

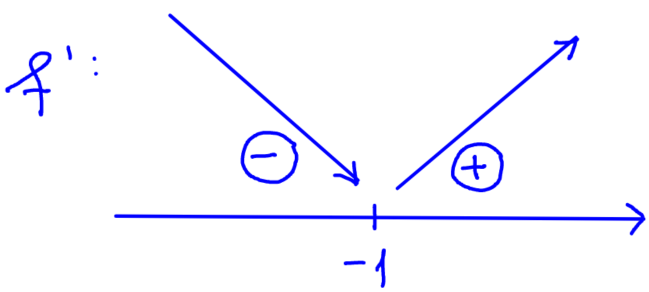
a) (10 točk) Določi in klasificiraj stacionarne točke.

5 $$f'(x) = \frac{1 \cdot e^{1-x} - x \cdot e^{1-x}(-1)}{(e^{1-x})^2} = \frac{e^{1-x}(1+x)}{(e^{1-x})^2} = \frac{1+x}{e^{1-x}} = 0$$

$$1 + x = 0$$

2 $\boxed{x = -1}$ lok. min. 3

b) (5 točk) Določi intervale naraščanja in padanja.



f pada na $(-\infty, -1)$ 5

f narašča na $(-1, \infty)$

c) (5 točk) Z uporabo L'Hospitalovega pravila izračunaj $\lim\limits_{x \to -\infty} f(x)$.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x}{e^{1-x}} \overset{L'H}{=} \lim_{x \to -\infty} \frac{1}{-e^{1-x}} = \underline{\underline{0}}$$

3 2

d) (5 točk) Določi normalo na graf funkcije f v točki $x = 1$.

2

$f(1) = 1$
$f'(1) = 2$

enačbo normale v točki $T(1,1)$:

$$y - f(1) = -\frac{1}{f'(1)}(x-1)$$

$$y - 1 = -\frac{1}{2}(x-1)$$

$$\boxed{y = -\frac{1}{2}x + \frac{3}{2}}$$

3

2. naloga (25 točk)

a) (10 točk) Z uporabo metode Per partes izračunaj nedoločeni integral

$$\int (x-2)e^x\, dx.$$

$$\int (x-2)e^x dx = (x-2)e^x - \int e^x dx =$$

3

$$\left[\begin{array}{l} u = x-2 \to du = dx \\ dv = e^x dx \to v = e^x \end{array}\right]$$

3

$$= (x-2)e^x - e^x + C$$
$$= e^x(x-2-1) + C$$
$$= e^x(x-3) + C$$

4

b) (15 točk) Z uvedbo primerne nove spremenljivke izračunaj določeni integral

$$\int_0^{\sqrt{\pi}} x\sin(x^2)\, dx.$$

$$\int_0^{\sqrt{\pi}} x\sin(x^2)dx = \int_0^{\pi} \sin(t)\frac{dt}{2} = \frac{1}{2}\int_0^{\pi}\sin(t)dt$$

3

$$t = x^2$$
$$dt = 2x dx$$
$$x dx = \frac{dt}{2}$$

3

$$= -\frac{1}{2}\cos(t)\Big|_0^{\pi} = 3$$

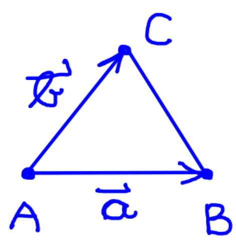
$$= -\frac{1}{2}\left(\cos(\pi) - \cos(0)\right) = 3$$

$$= -\frac{1}{2}(-1-1) = -\frac{1}{2}(-2) = \underline{\underline{1}} \quad 3$$

3. naloga (25 točk)

Dane so točke $A(0,2,2)$, $B(2,0,-1)$, $C(3,4,0)$, $D(2,-2,2)$ in vektor $\vec{v} = \begin{bmatrix} -3 \\ -1 \\ 10 \end{bmatrix}$.

a) (7 točk) Izračunaj ploščino ΔABC.



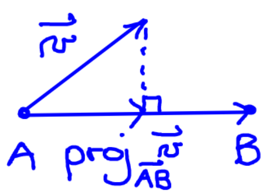
$$\vec{a} = \vec{AB} = \vec{r}_B - \vec{r}_A = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}$$

$$\vec{b} = \vec{AC} = \vec{r}_C - \vec{r}_A = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} \quad 2$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 10 \end{bmatrix} \quad 2$$

$$S_\Delta = \frac{|\vec{a} \times \vec{b}|}{2} = \frac{\sqrt{100+25+100}}{2} = \underline{\underline{\frac{15}{2}}} \quad 3$$

b) (6 točk) Poišči pravokotno projekcijo vektorja $\vec{v}$ na vektor $\vec{AB}$.



$$proj_{\vec{AB}}\vec{v} = \frac{\vec{AB} \cdot \vec{v}}{|AB|^2} \cdot \vec{AB} = 2$$

$$= \frac{2 \cdot (-3) + (-2)(-1) + (-3) \cdot 10}{\left(\sqrt{2^2 + (-2)^2 + (-3)^2}\right)^2} \begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix} =$$

2

$$= \frac{-34}{17}\begin{bmatrix}2\\-2\\-3\end{bmatrix} = -2\begin{bmatrix}2\\-2\\-3\end{bmatrix} = \begin{bmatrix}-4\\4\\6\end{bmatrix} \quad 2$$

c) (4 točke) Določi parameter t, tako da bosta vektorja $\vec{v}$ in $\vec{w} = \begin{bmatrix}2t\\2\\t\end{bmatrix}$ pravokotna.

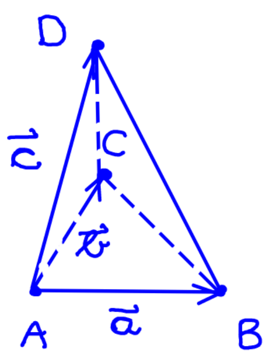
$$\vec{v} \cdot \vec{w} = 0$$

$$(-3)\cdot 2t + (-1)\cdot 2 + 10t = 0 \quad 3$$

$$4t = 2$$

$$\boxed{t = \frac{1}{2}} \quad 1$$

d) (8 točk) Izračunaj prostornino piramide $ABCD$.



$$\vec{c} = \overrightarrow{AD} = \vec{r}_D - \vec{r}_A = \begin{bmatrix}2\\-4\\0\end{bmatrix}$$

$$\vec{b} \times \vec{c} = \begin{bmatrix}3\\2\\-2\end{bmatrix} \times \begin{bmatrix}2\\-4\\0\end{bmatrix} = \begin{bmatrix}-8\\-4\\-16\end{bmatrix} \quad 2$$

$$(\vec{a}, \vec{b}, \vec{c}) = \vec{a} \cdot (\vec{b} \times c) =$$

$$= \begin{bmatrix}2\\-2\\-3\end{bmatrix} \cdot \begin{bmatrix}-8\\-4\\-16\end{bmatrix} = \quad 2$$

$$= 2(-8) + (-2)(-4) + (-3)(-16)$$

$$= -16 + 8 + 48 = 40 \quad 2$$

$$V_{piramide} = \frac{|(\vec{a}, \vec{b}, \vec{c})|}{6} = \frac{40}{6} = \underline{\underline{\frac{20}{3}}} \quad 2$$

4. naloga (25 točk)

Z uporabo Gaussove eliminacije poišči vse rešitve spodnjega sistema linearnih enačb.

$$\begin{array}{rcrcrcr} -x & - & 2y & + & z & = & -1 \\ & & 2x & + & 3y & = & 2 \\ & & y & - & 2z & = & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} -1 & -2 & 1 & -1 \\ 2 & 3 & 0 & 2 \\ 0 & 1 & -2 & 0 \end{array}\right] /\cdot 2 \; \oplus \sim \left[\begin{array}{ccc|c} -1 & -2 & 1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \oplus \sim$$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} -1 & -2 & 1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

15

$$-y + 2z = 0$$
$$\boxed{y = 2z}$$

$$-x - 2y + z = -1$$
$$x = -2y + z + 1$$
$$x = -2\cdot 2z + z + 1$$
$$\boxed{x = -3z + 1}$$

Rešitve:

$$\boxed{\begin{array}{ll} x = -3t+1 & \\ y = 2t & ;\; t \in \mathbb{R} \\ z = t & \end{array}}$$

10