1. Denoting $\mathbf{x} = [x, y]^{\mathsf{T}}$ find the general solutions to the system of differential equations $\dot{\mathbf{x}} = A\mathbf{x}$ in case A is the following matrix:

   (a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, (c) $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

   Use Octave to draw phase portraits (solution trajectories for several initial values) for each of the systems above. How do eigenvalues of the matrix A affect the behaviour of the solutions?

2. (a) Find the general solution $y(t)$ to the differential equation
   $$\ddot{y} + \dot{y} - 2y = 0.$$
   (b) Find a particular solution $y_p(t)$ to the differential equation
   $$\ddot{y} + \dot{y} - 2y = e^{-t}$$
   and using the result from (a) write down the general solution $y(t)$ to this equation.

   (c) Solve the initial value problem
   $$\ddot{y} + \dot{y} - 2y = e^{-t}, \, y(0) = 1, \, \dot{y}(0) = 2.$$

3. Find the solution $y(t)$ to the initial value problem
   $$\ddot{y} + 2\dot{y} + 5y = 5t + 7, \;\; y(0) = 2, \, y'(0) = 0.$$

4. Find the general solution $y(t)$ to the differential equation
   $$\dddot{y} + \ddot{y} - 2y = 2t^2.$$

5. The Van der Pol oscillator is a dynamical system with nonlinear damping satisfying this 2^{nd} order differential equation
   $$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0.$$
   (a) Rewrite this 2^{nd} order d.e as a system of two first order d.e.'s. Plot the phase diagrams for $\mu = 1$ and a few chosen initial values.

   (b) Find the stationary points of this first order system and compute the eigenvalues of Jacobi matrix in these stationary points (use Octave). What can you deduce about the stability of the stationary points?

   (c) Find the initial value $[x(0), \dot{x}(0)]^{\mathsf{T}}$ for which $y(0) = \dot{x}(0) = 0$ which describes a periodical solution. (Start with $x(0) = x_0 > 0$ and $\dot{x}(0) = 0$ and then find x_1 for which the trajectory intersects the half line x $\geq 0, y = 0$. The determines a function $f\colon (0, \infty) \to (0, \infty)$, $x_0 \mapsto x_1$. You need to find a fixed point of this map, i.e. solve the equation $f(x) = x$.)