Computational Complexity and Heuristic Programming, 2017/18
Written exam, 25 January 2018

All questions count equally. Literature, electronic and communication devices are not allowed. It is allowed to use up to 2 sheets of A4 format paper with handwritten notes. You can write your answers in Slovene. Duration: 90 minutes.
Oral exam for students who wish to improve their grade and have achieved at least 50% of points in written exam, will take place on Tuesday, 30 January 2018 at 11:00, in the office of Prof Robnik Šikonja (2nd floor, room 2.06).

1. We guess that the solution to the recurrence $T(n) = \sqrt{n} \cdot T\left(\sqrt{n}\right) + n$ is $T(n) = \theta(n \log\log n)$. Prove that the guess is correct or reject it.

2. Each of *n* customers gives a hat to a hat-check person at a restaurant. The hat-check person puts a hat on a white or black shelf and hands out a jetton with the colour of the shelf to the hat owner. When returning the hats to owners the hat-check person gives each customer a random hat from the shelf with the colour of the jetton. What is the expected number of customers who get back their own hat? Hint: use indicator random variables.

3. Given a set of *m* linear inequalities on *n* variables $x_1, x_2, \ldots, x_n$, the linear inequality feasibility problem asks whether there is a setting of the variables that simultaneously satisfies each of the inequalities. Show that if we have an algorithm for linear programming, we can use it to solve a linear-inequality feasibility problem. The number of variables and constraints that you use in the linear-programming problem should be polynomial in *n* and *m*.

4. A *clique* of size *k* is a fully connected (sub) graph consisting of *k* nodes. A *clique edge cover* is a set of cliques in G (fully connected subgraphs of G, possibly of different sizes) that together cover all the edges of G. A *minimum clique edge cover* is a clique edge cover that uses as few cliques as possible. We want to find the minimum clique edge cover existing in a given graph G. Justify your answers to the following problems.

   a) Propose a neighbourhood structure for local optimization approach to this problem.
   b) Propose features which could be used in penalization with guided local search.
   c) Suggest contents of tabu lists which could be useful in tabu search.