Computational topology

Lab work, 14^{th} week

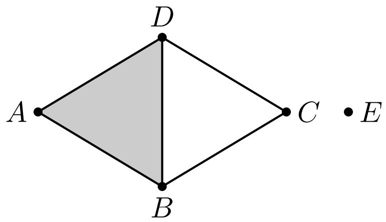
1. For a given simplicial complex K the function G is defined by the values in the following array.

σ	A	B	C	D	E	AB	AD	BC	BD	CD	ABD
$G(\sigma)$	3	2	0	3	2	6	4	1	4	1	5

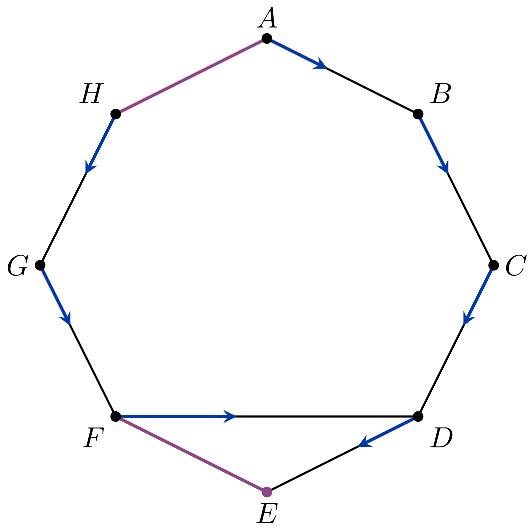
   (a) Show that G is discrete Morse function on K.

   (b) Determine the critical simplices and draw the corresponding vector field V_G.

   (c) Find all non-trivial gradient paths and use cancellation to obtain a new vector field with the minimal possible number of critical simplices.

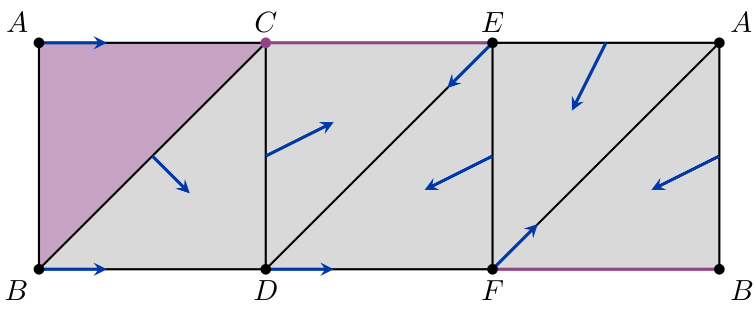


2. Use the given discrete vector field to compute the homology of this simplicial complex.

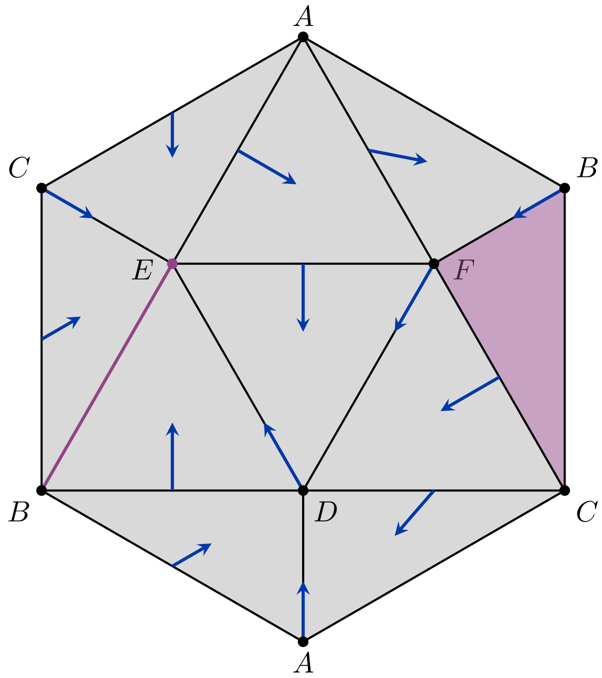


3. Using the triangulations given in the next three problems, construct an example of a discrete Morse function with a minimal number of critical simplices for the cylinder, the projective plane and the torus.

4. Use the given discrete vector field on the cylinder to compute its homology.



5. Use the given discrete vector field on the projective plane $\mathbb{R}P^2$ to compute its homology.



6. Use the given discrete vector field on the torus to compute its homology.