1. Evaluate the length of the curve K given by

$$\mathbf{p}(t) = [t^2 \cos t, t^2 \sin t]^\mathsf{T},\ t \in [0, 2\pi].$$

2. Evaluate the length of one of the arcs of the cycloid given by

$$\mathbf{q}(t) = [t - \sin t, 1 - \cos t]^\mathsf{T},\ t \in [0, 2\pi].$$

What is the area between the x-axis and one arc of the cycloid? (A *cycloid* is a curve traced by a point on the rim of a wheel rolling along the x-axis. The parametrisation given above is for a circle with radius $r = 1$.)

3. The *lemniscate* is a curve given in polar coordinates by

$$r(\phi) = a\sqrt{\cos 2\phi}.$$

Find a parametrisation of the lemniscate and evaluate the area of one of the regions enclosed by a loop.

4. **The circumference and the area of a planar polygon.** A polygon P in $\mathbb{R}^2$ is determined by a sequence of points $A_1, A_2, \ldots, A_k$. Write Octave functions `l = circumference(A)` and `pl = area(A)` that return the circumference and the area of the polygon P. The polygon is given by a matrix

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_k \\ y_1 & y_2 & \cdots & y_k \end{bmatrix}.$$

Additional task: Both functions should verify that the points $A_1, A_2, \ldots, A_k$ do indeed represent a polygon.

5. A surface in $\mathbb{R}^3$ is given by the implicit equation

$$\left(R - \sqrt{x^2 + y^2}\right)^2 + z^2 = r^2,$$

where $R > r$ are two positive numbers.

   (a) Verify that

$$\begin{aligned} x(\phi, \theta) &= (R + r\cos\theta)\cos\phi \\ y(\phi, \theta) &= (R + r\cos\theta)\sin\phi \\ z(\phi, \theta) &= r\sin\theta \end{aligned}$$

   is a parametrisation of this surface.

   (b) For $R = 2$ in $r = 1$ find the equation of the tangent plane at the point $T(1, \sqrt{3}, 1)$ using two different approaches: Using the implicit equation and using the parametrisation.