Topological Data Analysis
Lab work, 2nd week

1. Let $X_n = S^n \setminus \{(0,\dots,0,1),(0,\dots,0,-1)\} \subset \mathbb{R}^{n+1}$ and $Y_n = S^{n-1} \times (-1,1) \subset \mathbb{R}^{n+1}$. Draw X_n and Y_n for $n = 0,1,2$. Prove that X_2 and Y_2 are homeomorphic.

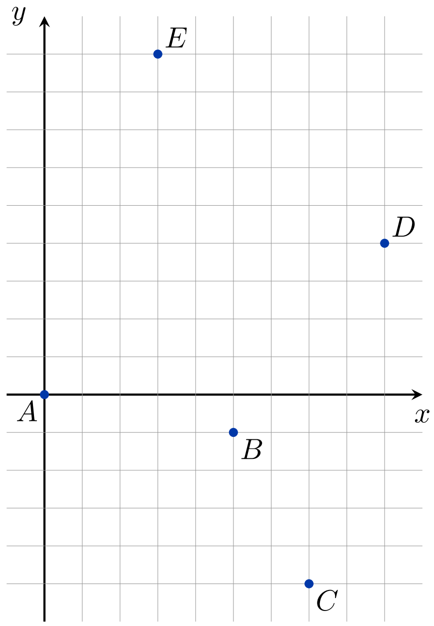
2. Draw $X_n = S^{n-1}$ and $Y_n = \mathbb{R}^n \setminus \{0\}$ for $n = 1,2$. Show that X_2 and Y_2 are homotopy equivalent.

3. Let $S = \{A(0,0), B(5,-1), C(7,-5), D(9,4), E(3,9)\} \subset \mathbb{R}^2$.

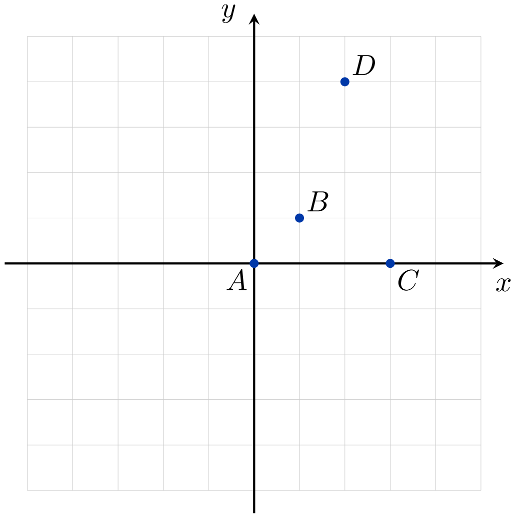
   (a) Construct the triangulations $\mathcal{T}_1$ and $\mathcal{T}_2$ of S using vertical line sweep from left to right and the horizontal line sweep upwards.

   (b) We can get the Delaunay triangulation on S by flipping certain edges. How many edge flips are necessary to produce a Delaunay triangulation from $\mathcal{T}_1$? From $\mathcal{T}_2$?

   (c) Draw the corresponding Voronoi diagram. Is it unique?



4. *Hermes messenger service, Ltd.* has distribution centres placed at $A(0,0)$, $B(1,1)$, $C(3,0)$ and $D(2,4)$. Divide the $[-5,5] \times [-5,5]$ square into service areas that ensure the fastest packet delivery.



Their competition, *Mercury post*, has the distribution centres located at $E(-4,-4)$, $F(4,-4)$ and $G(-2,4)$, but the center at E can only deliver within a 7 unit radius and the center at G only within a 6 unit radius. The center at F has more employees and uses bike messengers so they can deliver within an 10 unit radius. How should they split the service area?