Discrete Mathematics 13 (1975) 95.

NOTE

NEW GOSSIPS AND TELEPHONES

Walter KNÖDEL
University of Stuttgart, Stuttgart, Federal Republic of Germany

Received 10 May 1974
Revised 28 November 1974

Given n persons, each with a bit of information, wishing to distribute their information to one another in binary calls, each call taking a fixed time; how long must it take before each knows everything?

The answer is, for even n, $\lceil \log_2 n \rceil$, for odd n, $\lceil \log_2 n \rceil + 1$. That these bounds are best follows from the fact that it takes a sequence of s calls for the information from any one individual to reach 2^s people, so that time for at least $\lceil \log_2 n \rceil$ calls is necessary. In the odd case at least one more call is required because at least one participant will not distribute his information on the first round of calls.

The construction is trivial for $n = 2^k$. If one orders the participants, one can have $2j + 1$ call $2j + 2$ for each j. After the first round the odd participants separatedly can perform the 2^{k-1} procedure, and the even ones likewise, in the remaining rounds.

For odd n, one can choose $2^{\lfloor \log_2 n \rfloor}$ participants and have all the others call into them on the first and last rounds, otherwise proceeding as in the 2^k case. The procedure achieves the lower bound.

For general even n let $2j + 1$ call $2j + 2$ in the first round and $2j + 1$ call $2j + 2^r \bmod n$ in the rth round. It is easily seen by induction on r, that at the end of round r the persons 1 and 2^r each know all information from persons between them.

Acknowledgement

The author thanks the referee for essentially simplifying the proof and for his help in preparing the final draft.

Reference

B. Baker and R. Shostak, Gossips and telephones, Discrete Math. 2 (1972) 191–193.